Binary Representation & Number Systems

Lesson Objectives

- Binary Representation
- Distinguish among categories of numbers
- Describe Positional Notation
- Convert numbers in other bases to base 10
- Convert base-ten to numbers in other bases
- Describe the relationship between bases 2,8,16
- Explain the importance of computing of bases that are powers of 2

Data and Computers

Computers are **multimedia** devices, dealing with a vast array of information categories

Computers store, present, and help us modify

- Numbers
- Text
- Audio
- Images and graphics
- Video

Analog and Digital Information

Information can be represented in one of two ways: **analog** or **digital**

Analog data

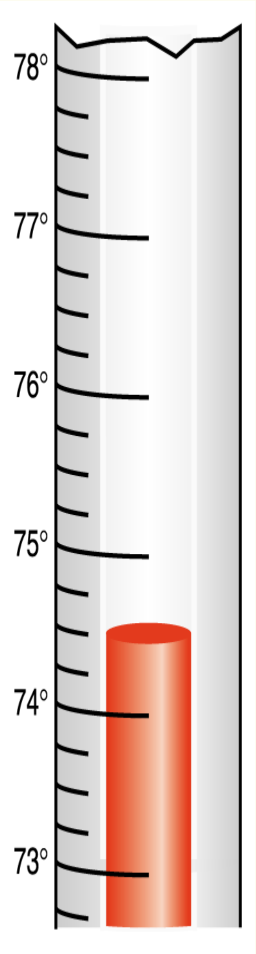
A continuous representation, analogous to the actual information it represents

Digital data

A discrete representation, breaking the information up into separate elements

Analog and Digital Information

Thermometer
is an
analog device



A mercury thermometer continually rises in direct proportion to the temperature

Analog and Digital Information

Computers cannot work well with analog data, so we digitize the data

Digitize

Breaking data into pieces and representing those pieces separately

Why do we use binary to represent digitized data?

Electronic Signals

Important facts about electronic signals

- An analog signal continually fluctuates in voltage up and down
- A digital signal has only a high or low state, corresponding to the two binary digits
- All electronic signals (both analog and digital) degrade as they move down a line
- The voltage of the signal fluctuates due to environmental effects

Binary Representations

One bit can be either 0 or 1

One bit can represent two things *(Why?)*

Two bits can represent four things *(Why?)*

How many things can three bits represent?

How many things can four bits represent?

How many things can eight bits represent?

Binary Representations

Counting with binary bits

1 Bit	2 Bits	3 Bits	4 Bits	5 Bits
0	00	000	0000	00000
1	01	001	0001	00001
	10	010	0010	00010
	11	011	0011	00011
		100	0100	00100
		101	0101	00101
		110	0110	00110
		111	0111	00111
			1000	01000
			1001	01001
			1010	01010
			1011	01011
			1100	01100
			1101	01101
			1110	01110
			1111	01111
				10000
				10001
				10010
				10011
				10100
				10101
				10110
				10111
				11000
				11001
				11010
				11011
				11100
				11101
				11110
				11111

Binary Representations

How many things can η bits *represent?*

Why?

What happens every time you increase the number of bits by one?

Binary System & Computers

- Modern computers are binary machines-By understanding number system, we understand how computing systems use binary number system to accomplish their task
- A digit in binary system is either 0 or1
- The binary values in a computer are encoded using voltage levels:
  - 0 is represented by a 0V signal (or low voltage)
  - 1 is represented by a high voltage signal.
- Bit – is a short expression for binary digit
- Byte – eight binary digits
- Word – a group of one or more bytes; the number of bits in a word is the word length in a computer
- We have 32-bit machine, 64-bit machine- affects the speed

Number Systems

- Number categories
  - Many categories: natural, negative, rational, irrational and many others important to mathematics but irrelevant to the understanding of computing
- Number – unit belonging to an abstract mathematical system and subject to specified laws of succession, addition and multiplication
  - **Natural number** is the number 0 or any other number obtained adding repeatedly 1 to this number.
  - A **negative number** is less than 0 and it is opposite in sign to a positive number.
  - An **integer** is any of positive or negative natural numbers
  - A **rational number** is an integer or the quotient of any two integer numbers
    - is a value that can be expressed as a fraction

Number Systems …

- The base of number system represents the number of digits that are used in the system. The digits always begin with 0 and continue through one less than the base
- Examples:
  - There are two digits in base two (0 and 1)
  - There are eight digits in base 8 (0 through 7)
  - There are 10 digits in base 10 (0 through 9)
- The base determines also what the position of the digits mean

Positional Notation

- It is a system of expressing numbers in which the digits are arranged in succession and, the position of each digit has a place value and the number is equal to the sum of the products of each digit by its place value
- Example:
  - Consider the number 954:
    - $9 * 10^2 + 5 * 10^1 + 4 * 10^0 = 954$
  - Polynomial representation - formal way of representing numbers, where X is the base of the number:
    - $9 * X^2 + 5 * X^1 + 4 * X^0$
- Formal representation – consider that the base of representation is B and the number has n digits, where d_i represents the digit in the ith position.
  - $d_n * B^{n-1} + d_{n-1} * B^{n-2} + \ldots + d_2B + d_1$
  - 642 is:

    $6_3 * 10^2 + 4_2 * 10 + 2_1 * 10^0$

Other bases

What if 642 has the base of 13?

$+ 6 \times 13^2 = 6 \times 169 = 1014$
$+ 4 \times 13^1 = 4 \times 13 = 52$
$+ 2 \times 13^0 = 2 \times 1 = 2$
$= 1068$ in base 10

642 in base 13 is equivalent to 1068 in base 10

Binary, Octal and Hexadecimal

- Decimal base has 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
- Binary is base 2 and has two digits (0 and 1)
- Octal is base 8 and has 8 digits (0, 1, 2, 3, 4, 5, 6, 7)
- Hexadecimal is base 16 and has 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

Converting Octal to Decimal

- What is the decimal equivalent of octal number 642?

$$
\begin{aligned}
& 6 \times 8^2 = 6 \times 64 = 384 \\
+\ & 4 \times 8^1 = 4 \times 8 = 32 \\
+\ & 2 \times 8^0 = 2 \times 1 = 2 \\
& = 418 \text{ in base } 10
\end{aligned}
$$

- Remember that octal base has only 8 digits (0, 1, 2, 3, 4, 5, 6, 7)

Converting Hexadecimal do Decimal

- What is the decimal equivalent of the hexadecimal number DEF?

$D \times 16^2 = 13 \times 256 = 3328$

$+ E \times 16^1 = 14 \times 16 = 224$

$+ F \times 16^0 = 15 \times 1 = 15$

$= 3567$ in base 10

- Remember that hexadecimal base has 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

MIT Student Effigy designed by Course 6 Students using Alphanumeric symbols. It is in front of Student Centre

Converting Binary to Decimal

- What is the equivalent decimal of the binary 10110 number?

$1 \times 2^4 = 1 \times 16 = 16$

$+ 0 \times 2^3 = 0 \times 8 = 0$

$+ 1 \times 2^2 = 1 \times 4 = 4$

$+ 1 \times 2^1 = 1 \times 2 = 2$

$+ 0 \times 2^0 = 0 \times 1 = 0$

= 22 in base 10

- Remember that binary base has only 2 digits (0, 1)

Review Question 1

What is octal number 11 in decimal representation?

A. 7
B. 8
C. 9
D. I don’t know …

Review Question 2

What is the decimal representation of binary number 1110?

A. 8
B. 14
C. 16
D. I don’t know …

Review Question 3

What is decimal representation of hexadecimal number FF?

A. 10
B. 255
C. 256
D. I don’t know …

Arithmetic in Binary

- The rules of arithmetic are analogous in other basis as in decimal base

Should read 1+1=0 with a carry of 1 similar to base 10 where 9 + 1 = 0 with a carry of 1 = 10

Addition			
1	1	0	0
+	+	+	+
1	0	1	0
10	1	1	0

-1 can be stated as 1 with a borrow of 1. Leading 1 we consider to be the sign, so 11 means -1

Subtraction			
1	1	0	0
-	-	-	-
1	0	1	0
0	1	11	0

Addition in Binary

- Base 2: 1+1 operation - the rightmost digit reverts to 0 and there is a carry into the next position to the left

```
  1 1 1        <- Carry Values
  0 1 0 1
+ 1 0 1 1
---------
1 0 0 0 0
```

- We can check if the answer is correct by converting the both operands in base 10, adding them and comparing the result

Subtracting in Binary

- The rules of the decimal base applies to binary as well. To be able to calculus 0-1, we have to “borrow one” from the next left digit.
- More precisely, we have to borrow one power of the base

```
     1 2
   0 2 0 2
   1 0 1 0
 - 0 1 1 1
 ---------
   0 0 1 1
```

- You can check if the result is correct by converting the operands in decimal and making the calculus.

Review Question 4

- Add 4 bit number 0100 with 0111. The answer is:

A. 1001

B. 1011

C. 1110

D. I don’t know …

Review Question 5

- Subtract 4 bit number 0100 from 1111. The answer is:

A. 1001
B. 1011
C. 1110
D. I don’t know …

Power of two Number Systems

- Binary and octal numbers have a very special relation between them: given a binary number, can be read in octal and given an octal number can be read in binary (i.e. have 753 in octal, in binary you have 111 101 011 by replacing each digit by its binary representation)
- Table represents counting in binary with octal and decimal representation

Binary	**Octal**	**Decimal**
000	**0**	**0**
001	**1**	**1**
010	**2**	**2**
011	**3**	**3**
100	**4**	**4**
101	**5**	**5**
110	**6**	**6**
111	**7**	**7**
1000	**10**	**8**
1001	**11**	**9**
1010	**12**	**10**

Converting Binary to Octal

- Start at the rightmost binary digit and mark the digits in groups of three
- Convert each group individually

10101011	10	101	011
	2	5	3

10101011 is 253 in base 8

- The reason that binary can be immediately converted in octal and vice-versa is because 8 is power of 2
- There is a similar relationship between binary and hexadecimal

Converting Binary to Hexadecimal

- Start at the rightmost binary digit and mark the digits in groups of four
- Convert each group individually

10101011 **1010** **1011**

A **B**

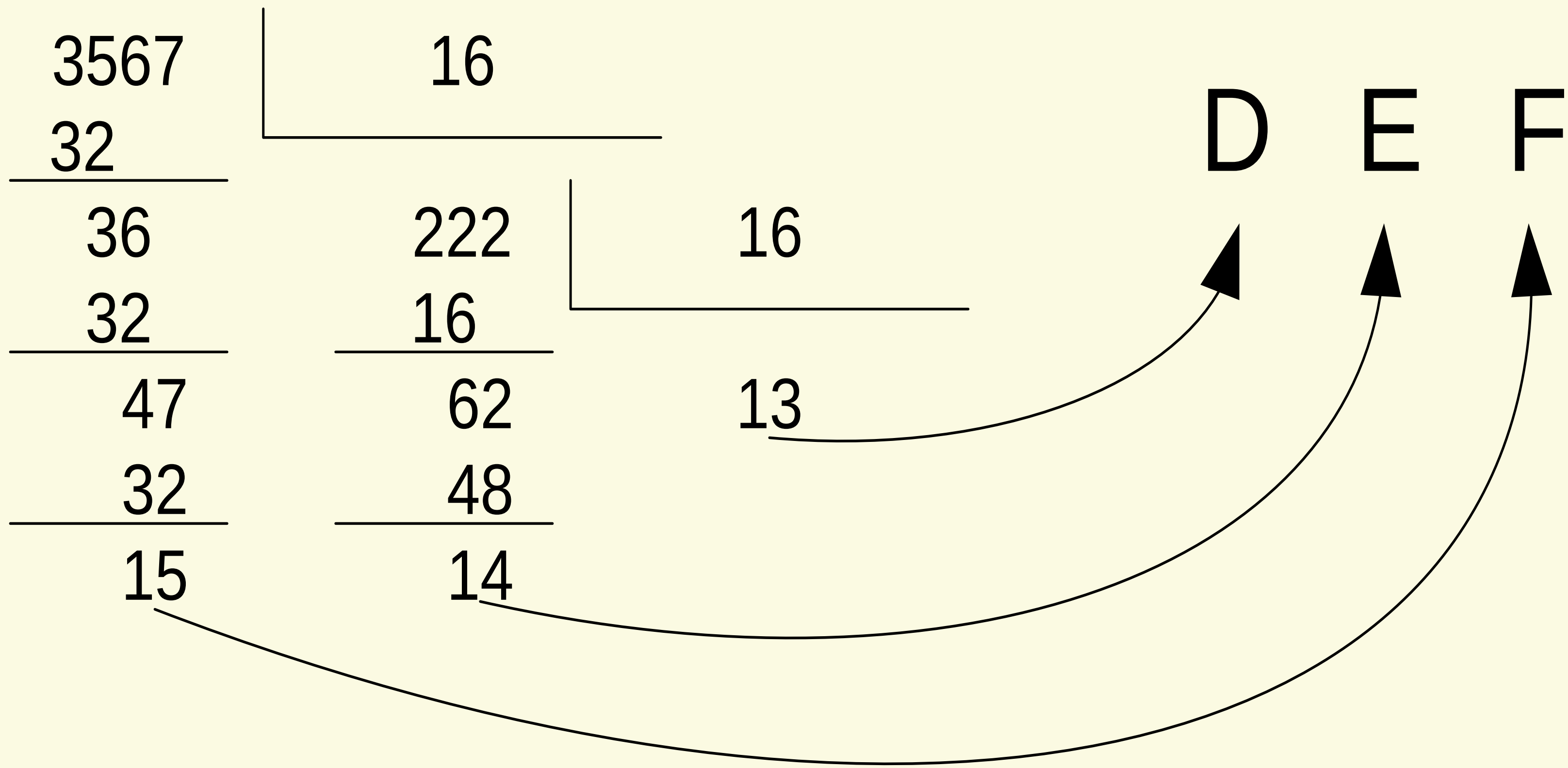
10101011 is AB in base 16

Converting Decimal to Other Bases

- Involves dividing by the base into which you convert the number
- Algorithm:
  - Dividing the number by the base you get a quotient and a reminder
  - While the quotient is ***not*** zero:
    - Divide the decimal number by the new base
    - Make the remainder the next digit to the left in the answer
    - Replace the original dividend with the quotient
- The base 10 number 3567 is what number in base 16?

Converting Decimal to Hexadecimal

- 3567 in decimal is DEF in hexadecimal



Review Question 6

Convert number 11001111 to hexadecimal. The answer is:

A. CF
B. BF
C. FC
D. I don’t know …

Review Question 7

Convert decimal number 375 to its octal representation. The answer is

A. 567
B. 765
C. 556
D. I don’t know …

Review Question 8

Convert decimal number 37 to its binary representation. The answer is:

A. 101001
B. 100101
C. 111000
D. I don’t know …